

Reply to the comment on “Incomplete equilibrium in long-range interacting systems” by Tsallis *et al.*

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After the rejection of their comment [arXiv:cond-mat/0609399v1] to our Phys. Rev. Lett. **97**, 100601 (2006), the Authors informed us that an extended version of their comment is going to be published in a different journal under the direct editorial responsibility of one of them. We then decided to make publicly available our formal reply, originally prepared for publication in Phys. Rev. Lett.

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In their comment [1], Tsallis *et al.* present three points which they claim to confute the conclusions of our work in Ref. [2]. Here we show that this is not the case.

An important issue about the quasi-stationary states (QSS's) displayed by isolated long-range interacting systems before relaxing to equilibrium is whether the QSS's survive to the perturbations introduced by a thermal reservoir (canonical QSS's). In order to answer this question, we proposed [3] an Hamiltonian setup in which the microcanonical conditions are recovered when the coupling constant between system and thermal bath vanishes. Indeed, we showed [3] that canonical QSS's exist, with a lifetime which decreases as the coupling strength increases. In [2] we discuss the statistics of the system energy fluctuations and, contrarily to what is claimed in [1], we do not “...extrapolate the conclusions for the canonical QSS's ... to the microcanonical ones”, where by definition the energy fluctuations are zero. Hence, the first criticism raised in [1] offers to the unaware Reader a false representation of our results and motivations.

On the other hand, we take here the opportunity of pointing out that the dynamical behavior during both microcanonical and canonical QSS's is such that the inter-particle correlation is negligible (see Fig. 1). Thus, in both cases the statistical mechanics description of the QSS's has to be based on the assumption of independence among elementary components and on the consequent application of the central limit theorem. This is consistent with our approach in [2], where we give compelling evidence of the applicability of Kinchin's derivation of statistical mechanics [4], once an appropriate estimation of the density of states $\omega(E)$ for the QSS's is given. It is also pertinent to add that the evidences we are presenting in Fig. 1 provide full justification to the use of the Vlasov theory for the QSS's, as it is done, e.g., in [5].

The statement [1] that the non-Gaussianity of the one-body angular momentum PDF “...excludes the Boltzmann-Gibbs exponential form as the energy distribution in full phase space Γ ...” is wrong. The Authors

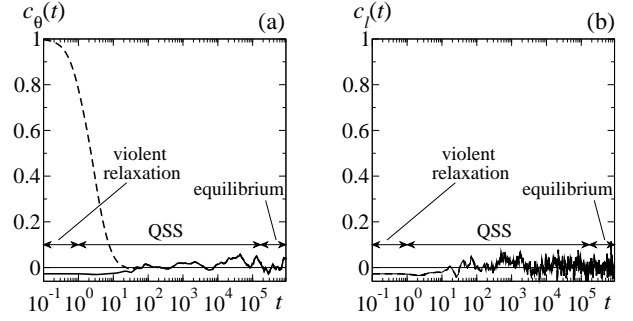


FIG. 1: Particle-particle correlation during a microcanonical simulation of the Hamiltonian system in [2] with $M = 5000$ particles. C_x ($x \equiv \theta, l$) is defined as $C_x \equiv (\langle x_i x_{i+I} \rangle - \langle x_i \rangle \langle x_{i+I} \rangle) / \sqrt{(\langle x_i^2 \rangle - \langle x_i \rangle^2)(\langle x_{i+I}^2 \rangle - \langle x_{i+I} \rangle^2)}$ (full line) and $C_x \equiv \langle x_i x_{i+I} \rangle / (\sqrt{\langle x_i^2 \rangle \langle x_{i+I}^2 \rangle})$ (dashed line), with $\langle \cdot \rangle \equiv (\sum_{i=1}^I \cdot) / I$ and $I < M/2$. Since at $t = 0$ all angles θ_i 's coincide (initial magnetization is 1), in (a) the dashed line starts from $c_\theta = 1$. In (b) the angular momenta l_i 's are initially uniformly distributed and the dashed and the full lines coincide for all times. In all simulations we observed that C_x during the QSS is not appreciably different than at equilibrium. The same is valid also for correlations of order higher than 2, even in canonical simulations or if the correlations are defined by averaging over different dynamical realizations.

of [1] are trivially proving that the PDF of a *microstate* $(l_i, \theta_i)_{i=1,2,\dots,M}$ is not proportional to $e^{-\beta H(l_i, \theta_i)}$, H being the Hamiltonian of the system with M particles. However, this does not imply that the *total energy* PDF must be different from $p(E) = \omega(E)e^{-\beta E}/Z$, where β is an inverse temperature. On the contrary, in force of the central limit theorem $p(E)$ still has the usual Boltzmann-Gibbs form [4], as we clearly verify in [2]. The nontrivial point is that $\omega(E)$ for the canonical QSS's is a *nonequilibrium* density of state which can be calculated by considering a submanifold of the Γ -space at constant magnetization [2].

In their final remark, the Authors of [1] propose a non-exponential fitting of our results for $\ln[p(E)/\omega(E)]$ after

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the introduction of an *ad hoc* energy-shift $E \mapsto E - 692$ for which they provide no explanation. A plot of the non-exponential function in [1] without this energy-shift does not agree at all with the data from the dynamical simulations. We also point out that by fitting $\ln[p(E)/\omega(E)]$ one implicitly assumes the validity of our calculation of $\omega(E)$, which is based on the fundamental thermodynamic

relation linking temperature to the Boltzmann expression for the entropy, $S \equiv k_B \ln[\omega(E)]$. As we explain in [2], to propose an alternative to the exponential weight $e^{-\beta E}$ without appropriately changing the determination of $\omega(E)$ is logically inconsistent.

In summary, we have shown that the statements in [1] either are wrong or do not apply to our results in [2].

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